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TECHNICAL REPORT ARLCB-TR-80031

ANALYSIS OF SHIFTS IN THE POINTS OF MAXIMUM DEFLECTION AND PERMANENT DEFLECTION FOR ELASTIC-PLASTIC BENDING OF UNSYMMETRICALLY LOADED BEAMS

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August 1980

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AMCMS No. 612105840011

DA Project No. 1L162105AH84

PRON No. 1A0217131A1A

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARLCB-TR-80031	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ANALYSIS OF SHIFTS IN THE POINTS OF MAXIMUM DEFLECTION AND PERMANENT DEFLECTION FOR ELASTIC-PLASTIC BENDING OF UNSYMMETRICALLY LOADED BEAMS		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. V. Milligan		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Benet Weapons Laboratory Watervliet Arsenal, Watervliet, NY 12189 DRDAR-LCB-TL		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 612105840011 DA Project No. 1L162105AH84 PRON No. 1A0217131A1A
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research and Development Command Large Caliber Weapon Systems Laboratory Dover, NJ 07801		12. REPORT DATE August 1980 13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Beams Plasticity Elastic-Plastic Analysis Bending Deflection		

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An analysis of elastic-plastic bending of unsymmetrically loaded beams is carried out using the method of virtual work. The results show that the maximum deflection under load occurs at quite some distance from the point of loading but moves toward the point of loading as the load approaches ultimate. The residual or permanent deflections occur at points different than those developed under load and are shifted further toward the load point. However, even at a load equal to 99.5% of the ultimate the maximum permanent deflection still occurs
(CONT'D)

Block 20. Abstract

at a considerable distance from the point of loading. The maximum strains (under load) and the permanent strains occur at the point where the load is applied.

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INTRODUCTION

In processing blank forgings for gun tube manufacturing, excessive permanent deformations are often developed. Straightening is then required to bring the tube component to within some manufacturing-specified tolerance. In some cases the maximum permanent deflection is quite close to the end of the tube which gives rise to an unsymmetrical distance-deflection curve similar to that shown in Figure 1(a). One might logically wonder if such an unsymmetrical deformation could be removed by inverting the tube and placing the straightening load at the point of maximum deflection as shown in Figure 1(b). The load would then be increased until elastic-plastic deformation was sufficiently large that when the load was removed the initial permanent deformation would be essentially eliminated. The objective of the present study was to gain a better appreciation of load-deflection relationships, which would hopefully give a better understanding of the straightening process for the case of three point, unsymmetrical bending.

Fig. 1(a)

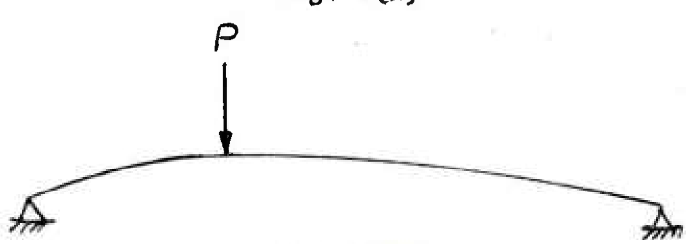


Fig. 1(b)

Figure 1. Sketch of unsymmetrically deformed beam.

APPROACH

This report is meant to be one in a series dealing with studies concerning aspects of elastic-plastic bending both from theoretical and experimental view points. The work presented here is theoretical and based on the method of virtual work. Without going into all of the mathematical details, which will be given elsewhere,[1] it will suffice to say that a solution is obtained by integrating the expression

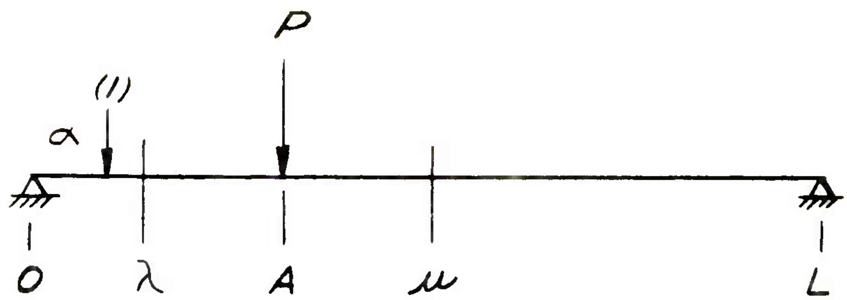
$$\int \frac{Mmdx}{EI}$$

where M = bending moment due to real load and

m = bending moment due to dummy load

over the length of the beam which is subdivided into four major sections as shown in Figure 2. First the integration is carried out from 0 to α, then from α to λ, which cover the elastic region on the left-hand side, then λ to A, and A to μ which cover the two elastic-plastic regions and finally from μ to L in the elastic region on the right-hand side of the beam. These deflection components give the deflection in the region 0 to λ (from the left support to the beginning of the elastic-plastic region relative to length). Next the so-called dummy load is placed between λ and A. An integration is made from o to λ in the elastic region, λ to α, α to A, and A to μ, all three integrals being in the two elastic-plastic regions and finally μ to L through the second elastic region. The dummy load is then placed in the region A to μ and the same type of integrations are made to determine the

[1] R. V. Milligan, "Load-Deflection Relationships for Simply Supported Beams Loaded Into the Plastic Region," to be published.



REGIONS

O TO λ ELASTIC
λ TO A ELASTIC/PLASTIC
A TO μ ELASTIC / PLASTIC
μ TO L ELASTIC

Figure 2. Sketch of beam showing elastic and elastic/plastic regions.

deflections in this region. Finally, the dummy load is positioned between μ and L to obtain the deflections in the elastic region on the right-hand side. In this way a continuous spectrum of deflections across the entire length of the beam is obtained. The value of M and m assume two different expressions depending on whether the section in question is to the right or left of the loads. The values λ and μ change as the load increases beyond the point of yielding up to the ultimate load. α is the distance from the left support to the point where the dummy load is applied - which is the point where the deflection is desired. Although the integrals are standard ones so as to be easily evaluated, there are a large number of them.

For determining the residual or permanent deflections, the beam is assumed to unload linearly, hence by ratio, the elastic deflection to be subtracted from the elastic-plastic deflection can be easily calculated by

$$\overset{*}{\delta} = \left(\frac{P_{ep}}{P_{el}}\right) \delta_{el}$$

where P_{ep} is the elastic-plastic load, P_{el} is the elastic load or at most the yield load, and δ_{el} is the elastic deflection corresponding to the particular load. The elastic deflections were calculated by using the double integration method. Of course, they can also be calculated by the method of Virtual Work, but the computer program is much longer.

RESULTS AND DISCUSSION

As an example, illustrating the results of this study, a problem for a segment of a 105 mm cannon tube was solved. The length was 216 inches and the load located 36 inches from the left end as shown in Figure 3(a). Figure 3(b) shows the cross section containing the Neutral axis and the plane of loading. The outside diameter was assumed constant. The material was assumed to be elastic-perfectly plastic with a stress-strain behavior as shown in Figure 3(c). The yield strength was taken as 160 ksi and modulus equal to 30 x 10^6 psi.

Figure 4 shows the elastic-plastic deflections vs length of beam for six different loads. The smallest corresponds to the yield load and the largest is 99.5 percent of the ultimate. Beyond this point the computer solution breaks down. One of the things to be noted is that the point of maximum deflection is located a considerable distance to the right of the load. Figure 5 is a plot of load vs distance to the point of maximum deflection. This figure shows that the maximum deflection shifts quite rapidly as the load approaches the ultimate but is still approximately 46 inches to the right of the load for P_{ep} equal to 99.5 percent of the ultimate load. The points of maximum deflection were obtained two different ways. First the increment of the do-loop was decreased and the point of maximum deflection simply read from the computer printout. A second method using splines[2] was

[2]R. V. Milligan, "Computer Analysis of Mechanical Test Data Using Cubic Splines," Transactions ISA, Vol. 17, No. 2, 1978, pp. 21-30.

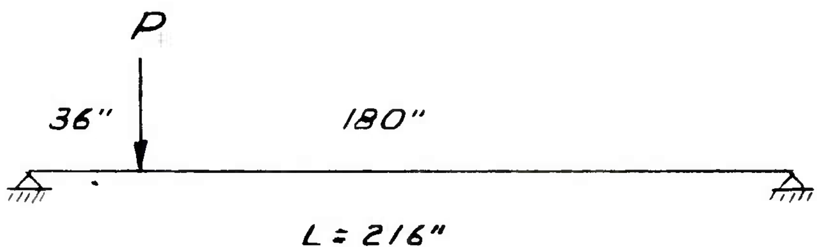


FIG 3a

Location of load

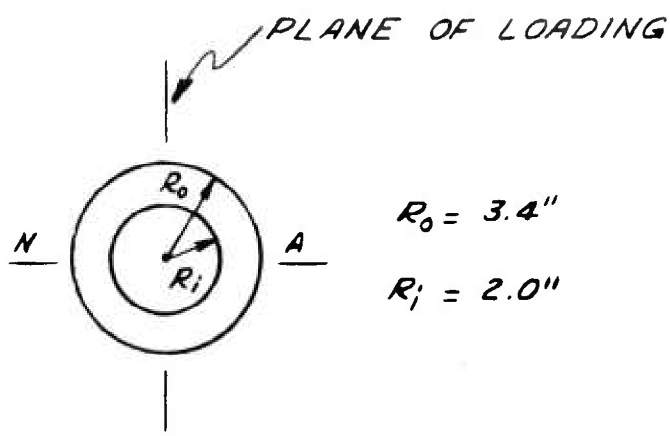


FIG. 3b

Dimensions of cross section of tube

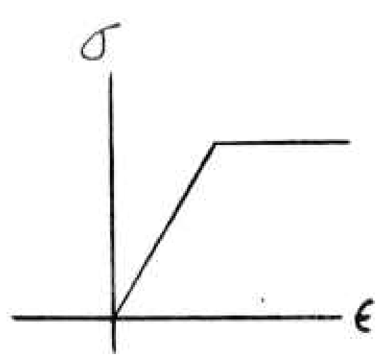


FIG. 3c

Idealized elastic perfectly-plastic behavior

FIGURE #3

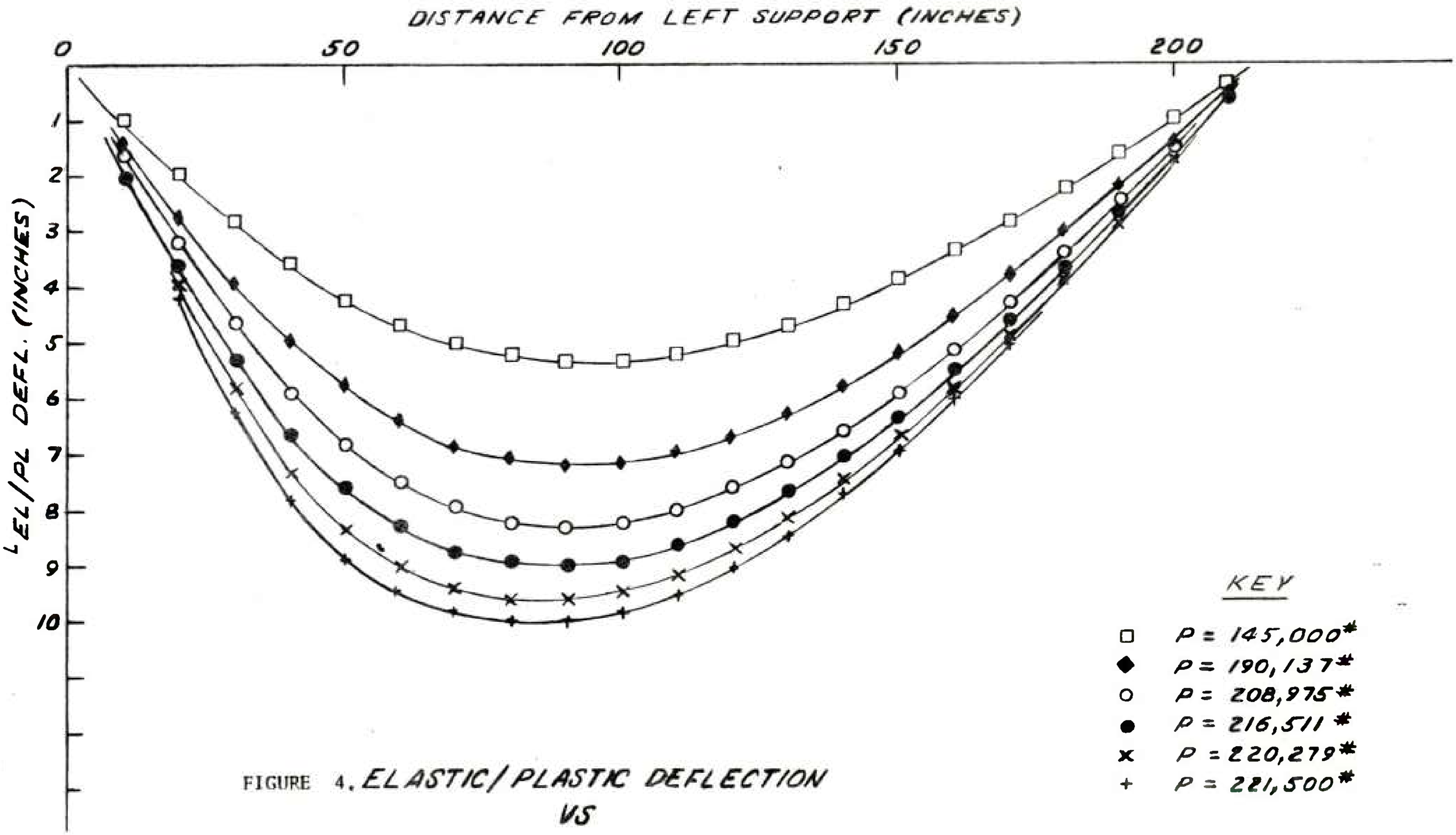


FIGURE 4. ELASTIC/PLASTIC DEFLECTION VS DISTANCE FROM LEFT SUPPORT

used. In this case the deflection vs distance data is fed into a spline routine where the first and second derivatives are calculated. From a Cal-Comp plot of the slope vs distance, a horizontal line is drawn from the point of zero slope on the vertical axis to intercept the curve. Then a line is projected downward to obtain the desired point on the horizontal axis. This is illustrated in Figure 6. Finally to complete this section a load vs maximum deflection curve is shown in Figure 7.

Figure 8 shows an exaggerated vertical scale plot of the permanent deflection vs distance from the left support. The interesting thing to note here is that these points of maximum deflection have shifted a considerable distance toward the load as compared with the case for the elastic-plastic deflections. Finally, Figure 9 shows a plot similar to Figure 5 except the abscissa represents the distance to the point of maximum permanent deflection. As before we can see that the curve flattens out as P_{ep} approaches P_{ult}. However, the distance to the point of maximum deflection is still nine inches to the right of the load application point.

Figure 10 is a plot of maximum permanent strain vs depth of the elastic-plastic interface. Curve A has the expanded scale on the right, while curve B has the scale on the left. One of the interesting points which should be pointed out is that the maximum strains occur on the outside fiber and at the point of loading. Therefore, the maximum permanent strains and the maximum permanent deflections do not occur at the same place along the length of the tube.

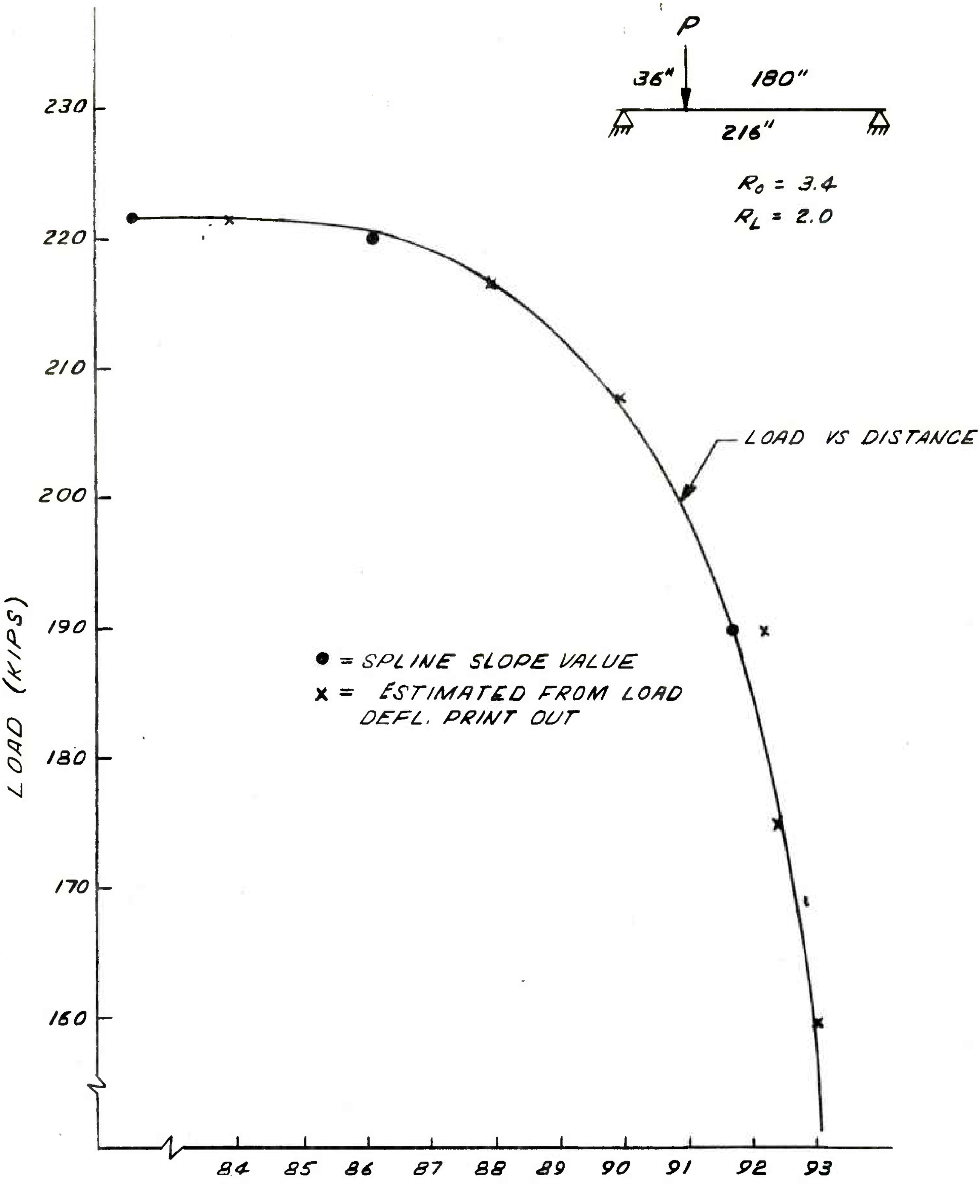


FIGURE 5
LOAD VS POINT ALONG LENGTH OF BEAM WHERE MAX DEFL. OCCURS (INCHES)

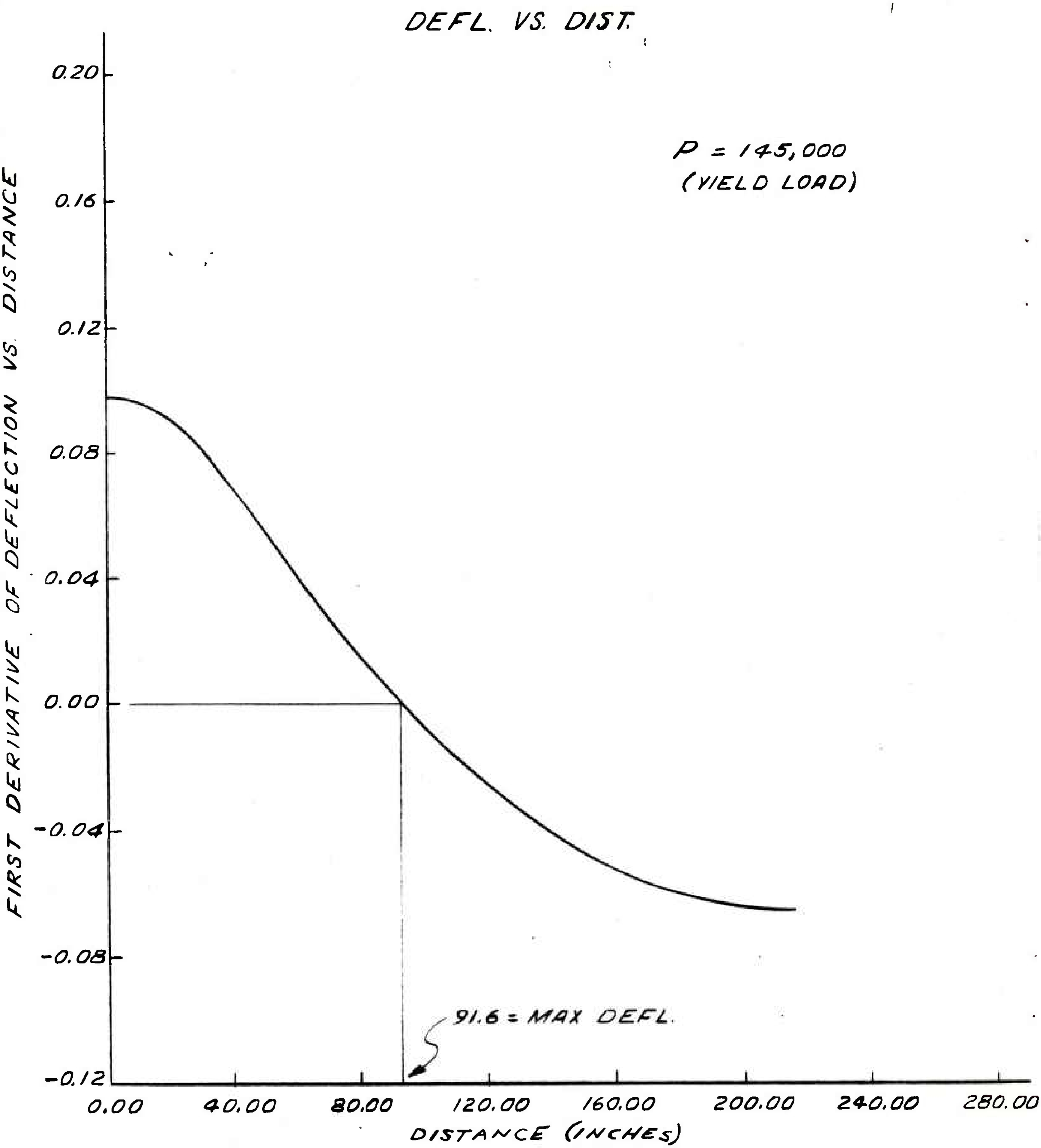


FIGURE 6. Deflection vs Distance

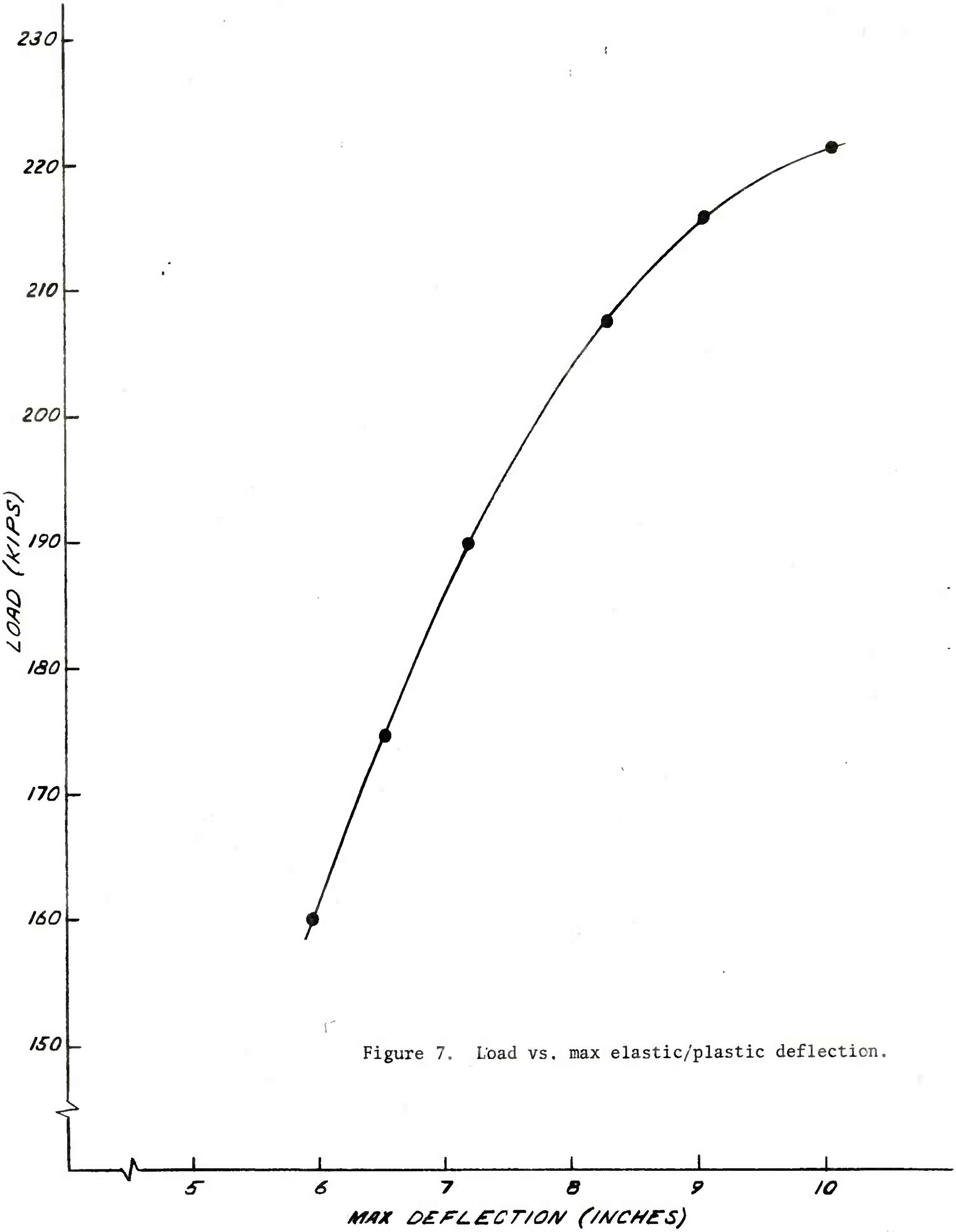


Figure 7. Load vs. max elastic/plastic deflection.

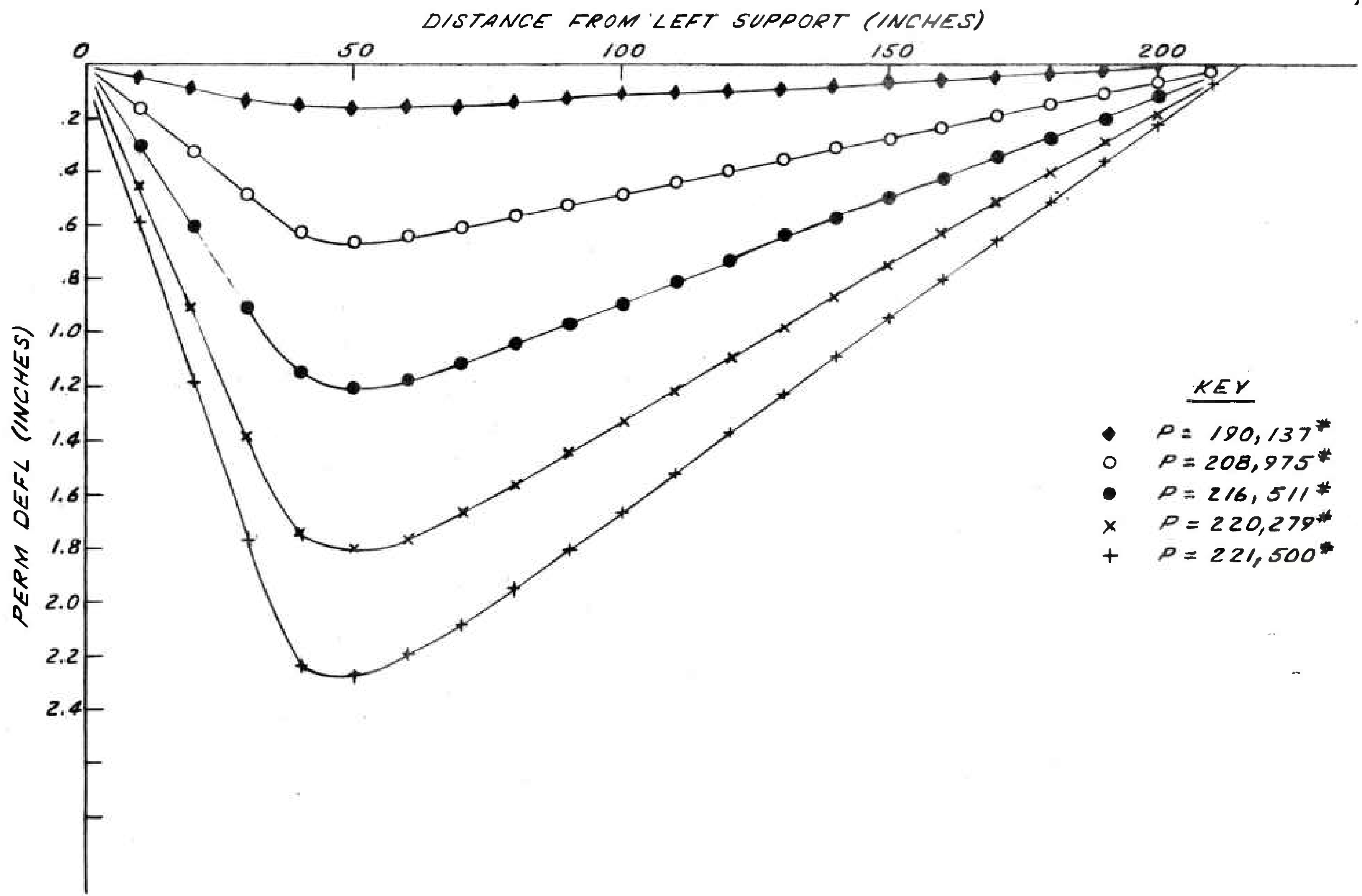


Figure 8. Permanent deflection vs. distance from left support.

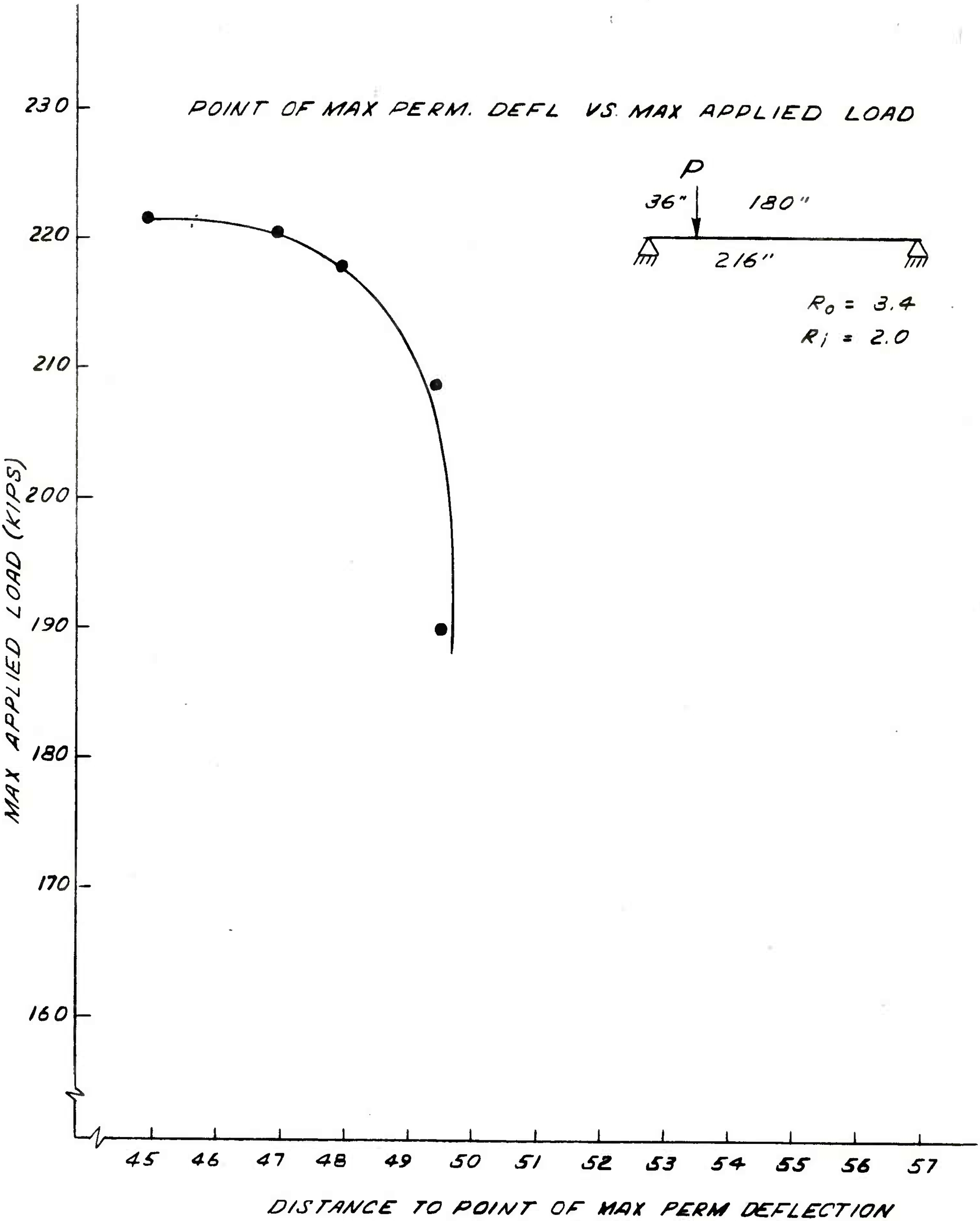
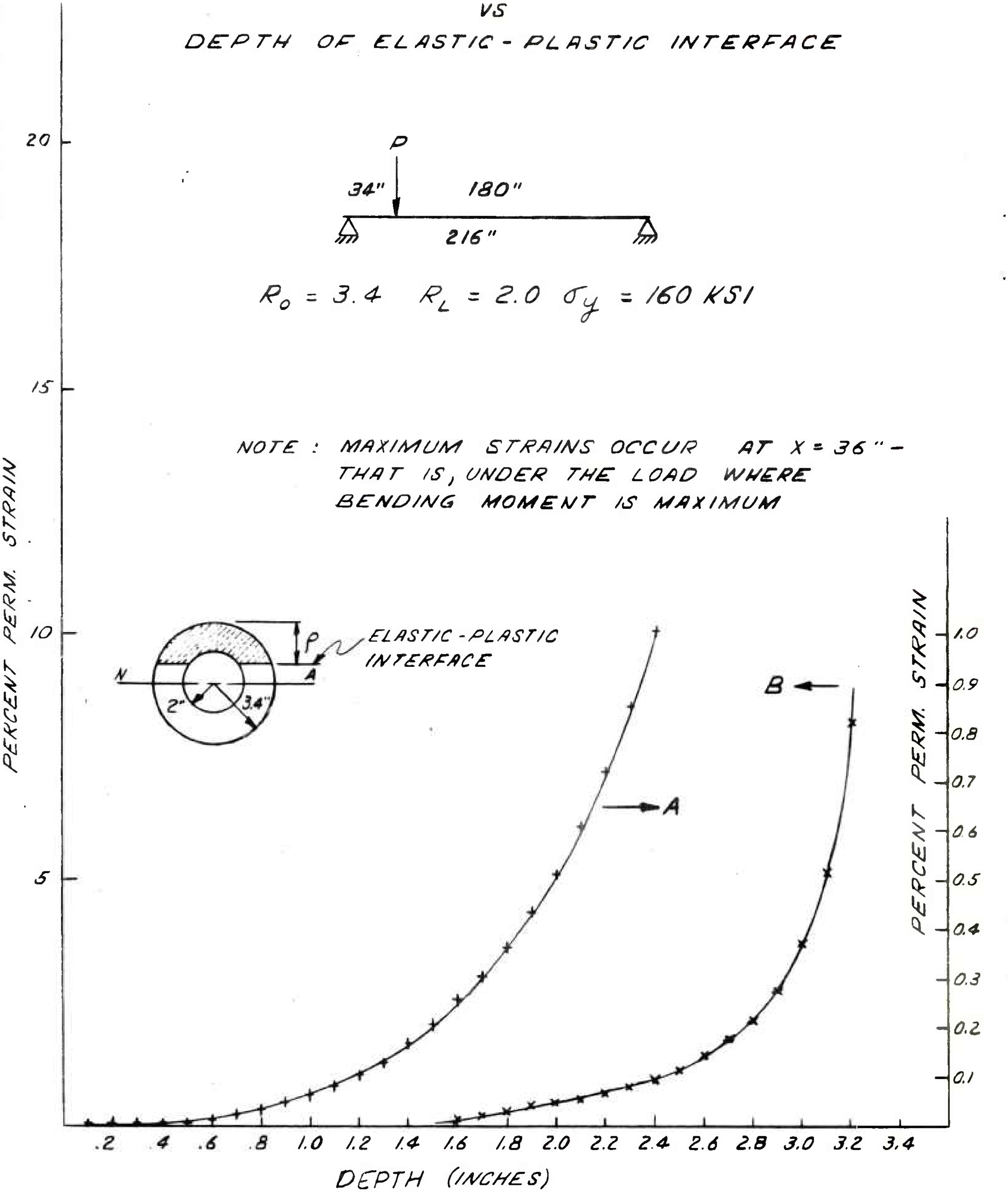


Figure 9. Maximum applied load vs. distance from left to support point of maximum permanent deflection.

FIGURE 10 - PERCENT PERMANENT STRAIN
VS
DEPTH OF ELASTIC - PLASTIC INTERFACE



These results are for the case of a constant outside diameter along the length of the tube. A tapered tube having a variable outside diameter would interject additional complications for determining these points of maximum deflection.

CONCLUSIONS

1. As in the elastic case, the point of maximum elastic-plastic deflection is located far to the right of the point of load application for a tube that is unsymmetrically loaded into the plastic region.

2. The points of maximum deflection for both the elastic-plastic and permanent cases shift rapidly towards the point of loading as the load approaches the ultimate.

3. For a load up to 99.5 percent of the ultimate load, the point of maximum permanent deformation is still a considerable distance to the right of the load point.

4. The point of maximum permanent deflection, along the length of the beam, does not coincide with the point where the maximum permanent strains occur.

REFERENCES

1. R. V. Milligan, "Load-Deflection Relationships for Simply Supported Beams Loaded Into the Plastic Region," to be published.

2. R. V. Milligan, "Computer Analysis of Mechanical Test Data Using Cubic Splines," Transactions ISA, Vol. 17, No. 2, 1978, pp. 21-30.

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